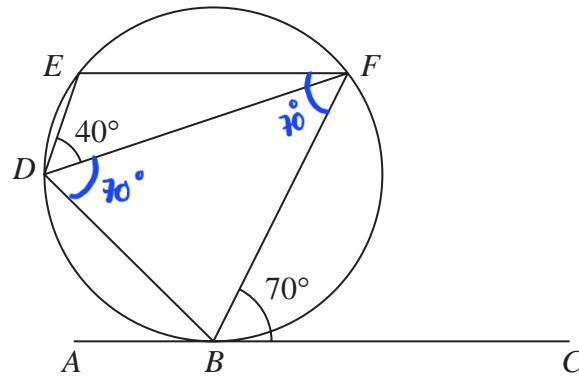**1**

Diagram **NOT** accurately drawn

B, *D*, *E* and *F* are points on a circle.
ABC is the tangent to the circle at *B*.

Angle *EDF* = 40°
Angle *FBC* = 70°

Prove that the tangent *ABC* is parallel to *EF*.
Give a reason for each stage of your working.

$\angle BDF = \angle FBC = 70°$ ①

(alternate segment theorem) ①

$\angle EFB = 180° - \angle EDB$

$= 180° - (40° + 70°)$

$= 180° - 110°$

$= 70°$ ①

(angles opposite to each other in a cyclic quadrilateral sums up to 180°)

∴ since $\angle EFB = 70°$ which is the same as $\angle FBC$,

line EF and line ABC are parallel. ①

($\angle EFB$ and $\angle FBC$ are alternate angles)

(Total for Question 1 is 4 marks)

2 The diagram shows a shaded shape *ABCD* made from a semicircle *ABC* and a right-angled triangle *ACD*.
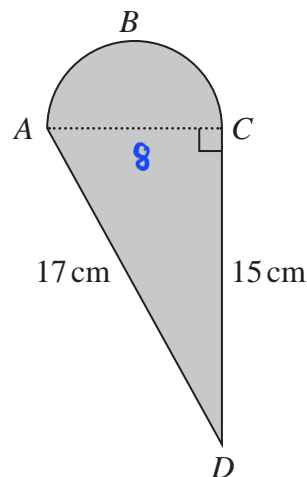
B

A ⌐⌐⌐⌐⌐⌐⌐⌐⌐ *C*
8

17 cm   15 cm

D

Diagram **NOT** accurately drawn

AC is the diameter of the semicircle *ABC*.

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

By using Pythagoras' Theorem:

$$AC^2 = AD^2 - CD^2$$
$$AC^2 = 17^2 - 15^2 \quad ①$$
$$AC = \sqrt{64}$$
$$= 8 \text{ cm} \quad ①$$

Length $ABC = \dfrac{\pi \times 8}{2} = 4\pi \quad ①$

Perimeter of shaded shape: $4\pi + 15 + 17 \quad ①$
$$= 44.6 \text{ cm}$$
$$①$$

44.6 .... cm

(Total for Question 2 is 5 marks)

3 Here is a sector, *AOB*, of a circle with centre *O* and angle *AOB* = *x*°

O

x°

26.193 ....

A   *B*
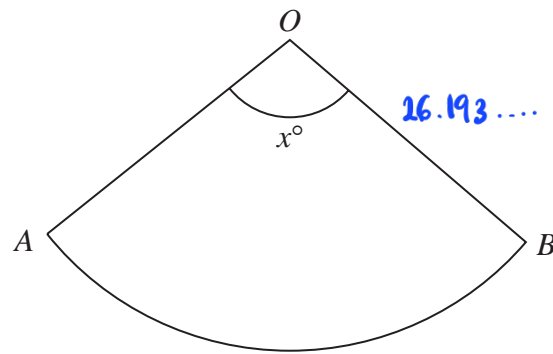
Diagram **NOT**
accurately drawn

The sector can form the curved surface of a cone by joining *OA* to *OB*.

O

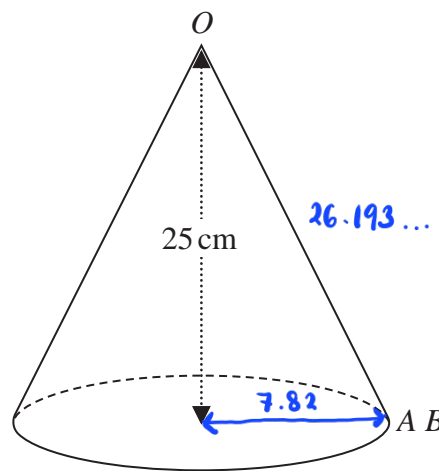25 cm

26.193 ...

7.82

A B

Diagram **NOT**
accurately drawn

The height of the cone is 25 cm.
The volume of the cone is 1600 cm³

Work out the value of *x*.
Give your answer correct to the nearest whole number.

Volume of Cone :

$$\frac{1}{3} \times \pi \times r^2 \times h$$

Finding radius of the cone :

$$\frac{1}{3} \times \pi \times r^2 \times 25 = 1600 \quad ①$$

$$\pi r^2 = \frac{1600}{25} \times 3$$
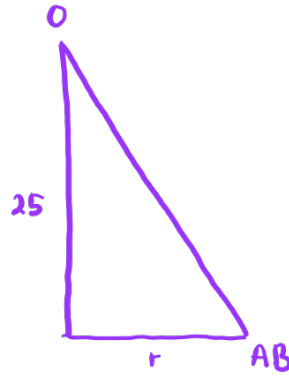
$$r^2 = \frac{192}{\pi}$$

$$r = \sqrt{61.115}$$

$$= 7.8176 ... \ cm \quad ①$$

By using Pythagoras' theorem :

$OA^2 = 25^2 + 7.8176\ldots^2$

$OA = \sqrt{686.1154\ldots}$

$= 26.193\ldots$ ①

circumference of the circle :

$2\pi r = 2 \times \pi \times 7.8176\ldots$

$= 49.1194\ldots$ ①

length of arc of the circle :

$2 \times \pi \times 26.193\ldots \times \dfrac{x}{360°} = 49.1194\ldots$ ①

$x = 107°$ ①

$x = $ ....107°........

(Total for Question 3 is 6 marks)

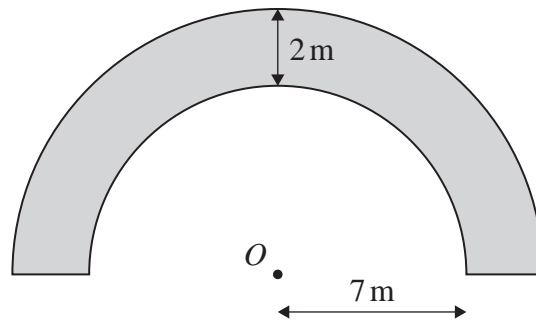**4** The region, shown shaded in the diagram, is a path.

Diagram **NOT** accurately drawn

2 m

O

7 m

The boundary of the path is formed by two semicircles, with the same centre *O*, and two straight lines.

The inner semicircle has a radius of 7 metres.
The path has a width of 2 metres.

Work out the perimeter of the path.
Give your answer correct to one decimal place.

Inner semicircle $= \frac{1}{2} \times 2\pi r$

$= \pi(7)$

$= 7\pi$ ①

Outer semicircle $= \frac{1}{2} \times 2\pi r$

$= \pi(9)$

$= 9\pi$

Perimeter $= 9\pi + 7\pi + 2(2)$ ①

$= 16\pi + 4$

$= 54.3 \text{ (1dp)}$

①

54.3 .... m

(Total for Question 4 is 3 marks)

5 *A*, *B* and *C* are points on a circle with centre *O*.
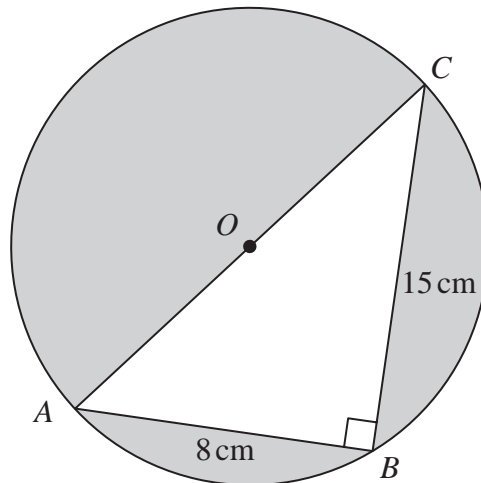
Diagram **NOT** accurately drawn

AOC is a diameter of the circle.

AB = 8 cm   *BC* = 15 cm

Angle *ABC* = 90°

Work out the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

Area of triangle $= \frac{1}{2} ab \sin C$

Area of triangle $= \frac{1}{2} \times 8 \times 15 \times \sin 90°$

$= 60$

$AC = \sqrt{8^2 + 15^2}$ ①

$= 17$ ①

radius of circle $= 17 \div 2 = 8.5$ cm

Area of circle $= \pi r^2$

$= \pi (8.5)^2$

$= 226.98$ ①

Area of shaded region $= 226.98 - 60$ ①

$= 166.98$

$= 167$ (3sf) ①

167

.... cm^2

(Total for Question 5 is 5 marks)

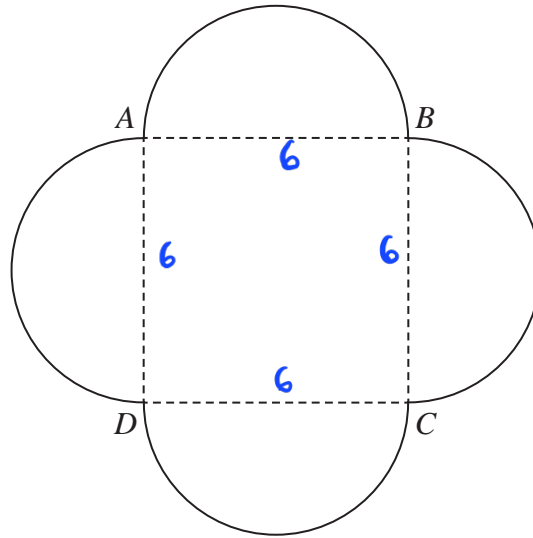**6** The diagram shows a shape made from a square *ABCD* and 4 identical semicircles.

Diagram **NOT** accurately drawn

As shown in the diagram, the semicircles have *AB*, *BC*, *CD* and *DA* as diameters.

The area of the square is $36\,\text{cm}^2$

Calculate the total area of the shape.
Give your answer correct to one decimal place.

Finding length of sides of ABCD :

$$x^2 = 36$$

$$x = 6 \text{ cm} \quad ①$$

∴ length of side of Square = diameter of semicircle = 6 cm

Area of each semicircle:

$$\frac{1}{2} \times \pi \times \left(\frac{6}{2}\right)^2 = \frac{9}{2}\pi \quad ①$$

Area of 4 Semicircle : $4 \times \frac{9}{2}\pi$

$$= 18\pi$$

Total area: area of square + area of 4 semicircle

$$= 36 + 18\pi \quad ①$$

$$= 92.5 \text{ cm}^2 \quad ①$$

92.5

.... cm^2

(Total for Question 6 is 4 marks)

7 The diagram shows a sector *AOB* of a circle with centre *O*

A

Diagram **NOT**
accurately drawn

6.08

5.2

67°

O   5.2 cm   *B*
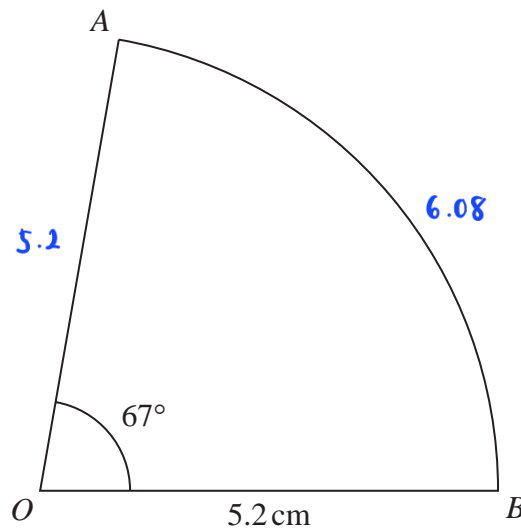
Angle *AOB* = 67°
OA = *OB* = 5.2 cm

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

Circumference of the whole circle:

$2 \times \pi \times 5.2 = \frac{52}{5} \pi$ ①

Arc length of the sector AOB:

$\frac{67}{360} \times \frac{52}{5} \pi = 6.08$ cm ①

Perimeter of the sector AOB:

$5.2 + 5.2 + 6.08 = 16.48$ ①

≈ 16.5 (3 s.f.)

16.5
.... cm

(Total for Question 7 is 3 marks)
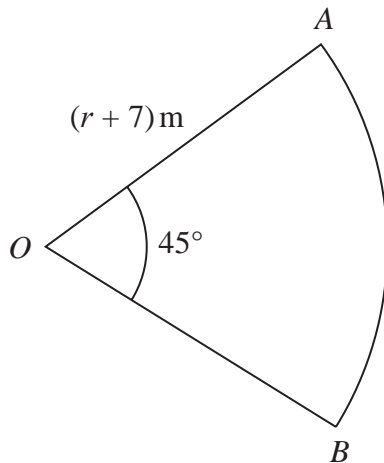
8

$(r + 7)\,$m

$45°$

O

A

B

Diagram **NOT**
accurately drawn

OAB is a sector **S** of a circle with centre O and radius $(r + 7)$ metres.
Angle $AOB = 45°$

A circle **C** has radius $(r − 2)$ metres.

The area of sector **S** is twice the area of circle **C**

Find the value of r
Show your working clearly.

$$\pi \times (r+7)^2 \times \frac{45}{360} = 2 \times \pi (r-2)^2$$

$$\pi \times (r^2 + 14r + 49) \times \frac{1}{8} = 2\pi \times (r^2 - 4r + 4)$$

$$r^2 + 14r + 49 = 16r^2 - 64r + 64$$

$$15r^2 - 78r + 15 = 0$$

$$5r^2 - 26r + 5 = 0$$

$$(5r - 1)(r - 5) = 0$$

$$r = \frac{1}{5} \quad \text{or} \quad r = 5$$

$r = 5$ only   since radius of circle

cannot be negative.

$r =$ **5** ....

(Total for Question 8 is 5 marks)

9 In the diagram, *ABC* is a right-angled triangle and *DEF* is a semicircular arc.
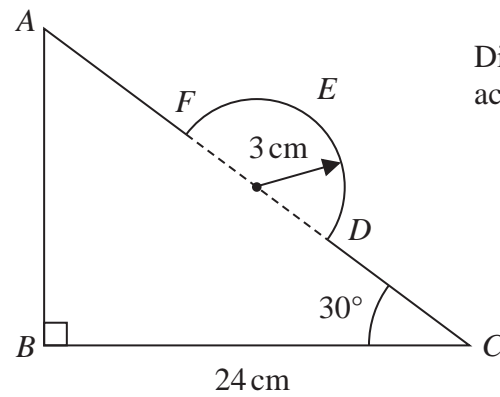
Diagram **NOT** accurately drawn

In triangle *ABC*

$$BC = 24\,cm \qquad \text{angle } ABC = 90° \qquad \text{angle } BCA = 30°$$

The points *D* and *F* lie on *AC* so that *DF* is the diameter of the semicircular arc *DEF*
The radius of the semicircular arc is 3 cm.

Work out the length of *AFEDC*
Give your answer correct to 2 significant figures.

$$\cos 30° = \frac{24}{AC} \ ①$$

$$AC = \frac{24}{\cos 30°} \ ① = 27.712\ldots$$

$$FED = \frac{1}{2} \times 2 \times \pi \times 3 \ ①$$

$$= 3\pi = 9.424\ldots$$

$$AFEDC = 27.712 - 3 - 3 + 9.424 \ ①$$

$$= 31 \ ①$$

31

.... cm

(Total for Question 9 is 5 marks)

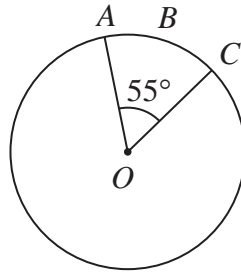**10** The diagram shows a circle with centre O

A B
C
55°
O

Diagram **NOT** accurately drawn

A, B and C are points on the circle so that the length of the arc ABC is 5 cm.

Given that angle $AOC = 55°$

work out the area of the circle.
Give your answer correct to one decimal place.

$$\frac{55}{360} \times 2 \times \pi \times r = 5 \quad \text{(1)}$$

$$r = \frac{5 \times 360}{2 \times \pi \times 55} \quad \text{(1)}$$

$$\approx 5.2$$

$$\text{Area} = \pi \times 5.2^2 \quad \text{(1)}$$

$$\approx 85.2$$
$$\text{(1)}$$

85.2

.... cm^2

(Total for Question 10 is 4 marks)
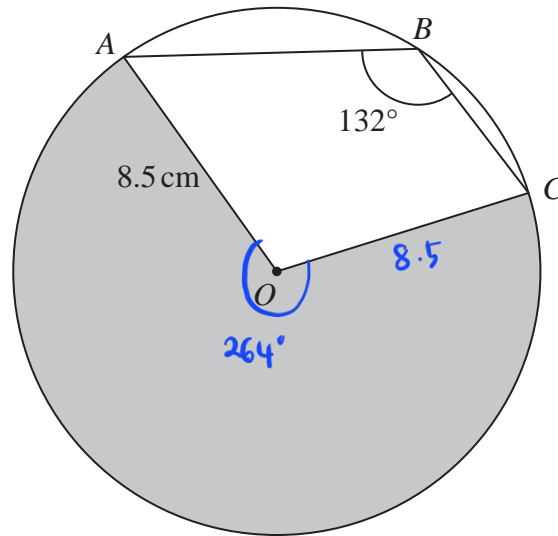
11 *A*, *B* and *C* are points on a circle, centre *O*

Diagram **NOT** accurately drawn

The radius of the circle is 8.5 cm
Angle *ABC* = 132°

Work out the perimeter of the shaded sector *AOC*
Give your answer correct to 3 significant figures.

$\angle AOC = 132° \times 2 = 264°$ ①

$\dfrac{264}{360} \times 2 \times \pi \times 8.5 = 39.1...$ ①

Perimeter : 39.1.. + 8.5 + 8.5

≈ 56.2 ①
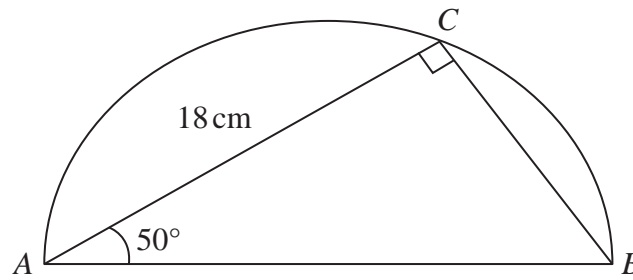
56.2

.... cm

(Total for Question 11 is 3 marks)

12 The diagram shows a triangle *ABC* inside a semicircle.

Diagram **NOT**
accurately drawn

C

18 cm

50°

A B

A, *B* and *C* are points on the semicircle.

AB is the diameter of the semicircle.

Angle *ACB* = 90°
Angle *BAC* = 50°
AC = 18 cm

Work out the perimeter of the semicircle.
Give your answer correct to 2 significant figures.

$$\cos 50° = \frac{18}{AB} \quad ①$$

$$AB = \frac{18}{\cos 50°} \quad ①$$

$$= 28.0030...$$

$$\frac{1}{2} \times π \times 28.0030... = 43.9... \quad ①$$

$$28.0030... + 43.9.... \quad ①$$

$$= 71.9900...$$

$$≈ 72 \quad ①$$

72
.... cm

(Total for Question 12 is 5 marks)

13 The diagram shows two circles with centre *O* and a regular pentagon *ABCDE*
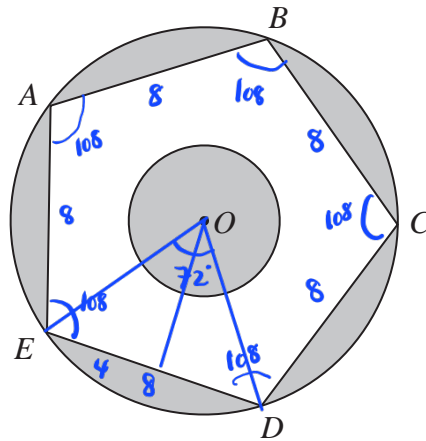

Diagram **NOT** accurately drawn

A, *B*, *C*, *D* and *E* are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

$$\text{shaded area} = \text{unshaded area}$$

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

Pentagon
$$\text{angle} = \frac{180 \times 3}{5} = 108°$$

$$\text{angle } EOD = 180 - 54 - 54$$
$$= 72°$$

height of triangle , $\tan 54 = \dfrac{\text{height}}{4}$

$$= 4 \tan 54 = 5.505 \ldots ①$$

$$\frac{\text{length } OE}{\sin 54°} = \frac{8}{\sin 72}$$

$$OE = \frac{8 \sin 54}{\sin 72} = 6.805 \ldots = \text{radius of large circle}$$

Area of whole diagram $= \pi \times 6.805^2 = 145.489 \ldots ①$

Area of pentagon $= 5 \times \dfrac{1}{2} \times 8 \times 5.505 \ldots = 110.11 \; ①$

Shaded area = unshaded area

$145 \cdot 489 - 110 \cdot 11 + \pi r^2 = 110 \cdot 11 - \pi r^2$ (1)

$2 \pi r^2 = 74 \cdot 731 \ldots$ (1)

$r^2 = 11 \cdot 89 \ldots$

$r = 3 \cdot 45 \quad (3 \text{ s.f.})$ (1)

3·45

.... cm

(Total for Question 13 is 6 marks)